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Coefficients of the Second Viscosity in Thin Liquid-Helium Films

by

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## Coefficients of the second viscosity in thin liquid-helium films

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The coefficients of the second viscosity in thin liquid-helium films are evaluated explicitly as a function of temperature (T) via interactions between elementary excitations which include the anomalous excitation spectrum. For T  $\geq 0.3$  K the coefficients behave like those of the bulk liquid helium, while for T  $\leq 0.3$  K the temperature variations of four coefficients of the second viscosity show a T<sup>-1</sup>-dependence, which is due to the three-phonon processes (3PP) originating from the anomalous excitation spectrum of two-dimensional liquid helium at low momenta and low temperatures. The T<sup>-1</sup>-dependence also appears in the first viscosity of bulk and thin liquid helium, which is based on the 3PP.

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#### 1. Introduction

Recently, the temperature variations of various sound propagations have been an important subject in the studies of liquid  ${}^3\text{He}$ ,  ${}^1$   ${}^4\text{He}$ ,  ${}^2$   ${}^3\text{He}$ - ${}^4\text{He}$  mixtures  ${}^3$  and solutions of  ${}^3\text{He}$  in liquid  ${}^4\text{He}$ ,  ${}^4$  because their properties are closely related to the exitation spectrum of liquid  ${}^3\text{He}$  and  ${}^4\text{he}$ , which can determine the transport coefficients  ${}^5$  through the kinetic equations. It is well known that at low temperatures and low pressures the thermal properties of superfluid helium are not determined mainly by the rotons but rather by the low-momentum acoustic phonons. These phonons do not possess the normal dispersion relation, but instead exhibit an anomalous excitation spectrum. In fact, these two cases are quite different in microscopic processes. The former contributes to four-phonon processes (4PP) while the latter is governed by three-phonon processes (3PP).

Recently, Andreev and Khalatnikov have evaluated the temperature variation of first sound, and Singh and Prakash have used the retarded single-particle Green's function for a weakly-interacting Bose gas to obtain the first sound by using the wrong normal dispersion relation. However, in a recent paper we have developed the Landau-type elementary excitation spectrum, which is anomalous phonon-like at low momenta and roton-like at high momenta in two- and three-dimensional liquid helium. Starting with this elementary excitation, we have successfully derived and explained not only the various sounds and sound attenuations, but also thermal conductivity and viscosity in partly one-, two- and three-dimensional liquid whe.

Regarding the evaluation of first and second sound, <sup>13</sup> we have used a new approach, which takes into account a collision term in the Boltzmann equation, and have obtained first and second sound simultaneously. We reported that the second sound in thin helium films is approximately 2<sup>-14</sup> times the first sound

velocity in the low-temperature and low-frequency limit. At low frequencies such that  $\omega_S \tau << 1$ , where  $\omega_S$  is the sound frequency and  $\tau$  is the characteristic time, we can make use of a hydrodynamical approach to sound propagation. However, for the opposite case of  $\omega_S \tau >> 1$ , it is better to use the kinetic equations. In this collisionless region the first and second sound obtained in superfluid hydrodynamic equations are involved in the attenuation coefficients, which contain the four coefficients of second viscosity, i.e.,  $\zeta_1$ ,  $\zeta_2$ ,  $\zeta_3$  and  $\zeta_4$ . These four coefficients play a very important role in the investigation of sound attenuation in bulk liquid helium. In the bulk case, the attenuation of second sound depends on all four viscosity coefficients together. However, the term which generally contains the thermal conductivity strongly affects the attenuation.

Recently, we have adopted the Landau and Khalatnikov theory to derive the thermal conductivity  $^{11}$  and viscosity  $^{12}$  for three ranges of temperature,  $T \le 0.3$  K, 0.3 K  $\le T \le 0.8$  K and  $T \ge 0.8$  K, in which the scattering depends on the nature of interactions between elementary excitations. For  $T \le 0.3$  K it is important to note that the 3PP do not affect the thermal conductivity but cause the viscosity to have a  $T^{-1}$ -dependence, which was not proven by Landau and Khalatnikov's results. The 4PP and phonon-roton scattering govern the whole mechanism in the range of temperature 0.3 K  $\le T \le 0.8$  K, and for  $T \ge 0.8$  K the 5PP and the phonon-roton scattering contribute mostly to the transport coefficients.

The main purpose of the present paper is to evaluate the four coefficients  $\zeta_1$ ,  $\zeta_2$ ,  $\zeta_3$  and  $\zeta_4$  of the second viscosity as a function of temperature by solving the superfluid hydrodynamics for the above temperature ranges through the theory of the kinetic phenomena developed by Khalatnikov,  $^{14}$  and especially for T  $\leq$  0.3 K we will investigate the temperature variation of

the four viscosity coefficients. Throughout this paper we will use the two-dimensional elementary excitation spectrum obtained by the microscopic ring-diagram approximation,  $^{8}$ 

$$E(p) = c_0 p[1 + \gamma p^2 - \delta p^4 + ...] , \qquad (1.1)$$

$$E(P) = \Delta + \frac{(P-P_0)^2}{2\mu_0}$$
, (1.2)

where p and P are phonon and roton momenta, respectively,  $c_0$  is the sound velocity at absolute zero temperature,  $\gamma$  and  $\delta$  are positive constants which can be determined by the potential parameters, and  $\Delta$ ,  $P_0$  and  $\mu_0$  are the roton parameters. Here, we have adopted a soft potential with a Lennard-Jones-type tail which helps to make a smooth connection between the attractive part and a soft repulsive core:

$$\phi(r) = \begin{cases} V_0 & , & r \leq a \\ & & \\ E_0[(\frac{a}{r})^{12} - (\frac{a}{r})^6] & , & r \geq a \end{cases}$$
 (1.3)

In this paper we define the liquid helium film as two-dimensional -less than three atomic layers, namely one statistical layer of 3.6 Å -- and
neglect substrate effects. In Sec. 2, to obtain the temperature variation of
the viscosity coefficients in various processes, we will investigate the
absorption and emission processes between elementary excitations through the
collision integral equations, and we will solve the superfluid hydrodynamic
equations to obtain four coefficients of the second viscosity in Sec. 3.

Finally, in Sec. 4 we will give the results and discussion together with the thermodynamic properties in terms of a table and graphs.

### 2. Absorption and emission of phonons and rotons.

To describe the interactions between elementary excitations which cause liquid helium to make a transition from one state to another, we can consider generally three collision processes: (a) emission or absorption of phonons, (b) transformation of phonons into rotons and vice versa, and (c) emission or absorption of rotons. In addition, we have the 3PP, 4PP and 5PP. In the case of (c) the energy of one roton would have to be at least  $2\Delta$ , such that it can decay into two rotons each with energy of about  $\Delta$ . However, this kind of three-roton process is highly improbable, so that we may neglect it. Therefore, from now on, we only take into account the 3PP and 5PP and transformations of phonons into rotons and vice versa.

We consider first the 3PP  $(\vec{p}_2 + \vec{p} + \vec{p}_1)$ . Since the total number of phonons traveling in a given direction is changed by the small-angle 3PP, the distribution function, which depends on the chemical potential  $\alpha'$ , can be written as

$$n = \left[ \exp(\alpha' + pc) / k_B T - 1 \right]^{-1}$$
 (2.2)

The difference between Eq. (2.2) and the equilibrium distribution function  $n_0$  can be expanded in a power series to give

$$\delta n = n - n_0 = -n_0(n_0 + 1) \alpha'$$
 (2.3)

The collision integral  $J_{3PP}(n)$  and the kinetic coefficient  $\Gamma_{3PP}$  are related to each other through

$$\frac{1}{k_B T} \int J_{3PP}(n) \frac{d\vec{p}}{(2\pi k)^2} - \alpha' \Gamma_{3PP} , \qquad (2.4)$$

and the collision integral  $J_{\mbox{\footnotesize 3PP}}(n)$  obtained in the second quantization method  $^{11}$  is given by

$$J_{3PP}(n) = -\frac{\pi c_0}{2 \aleph \rho_0} (u+1)^2 \int p_1 p_2 p \delta n (n_{p_10} - n_{p_20}) \delta (E_f - E_i) \frac{d^2 p}{(2\pi \aleph)^2} , \quad (2.5)$$

where u is Grüneisen's constant, and n represents the equilibrium  $p_{i0}$  distribution function for the phonons with momentum  $p_i$ . Performing the integration for Eq. (2.5) over momentum space, we obtain

$$\int J_{3PP}(n)pdp = \frac{2!3!\zeta(2)\zeta(3)(u+1)^2}{8\pi h^3 \rho_0} \alpha' \left(\frac{k_B^T}{c_0}\right)^6 \qquad (2.6)$$

Comparing Eqs. (2.4) and (2.6), we can easily obtain the kinetic coefficient  $\Gamma_{\rm 3PP}$  for the 3PP as

$$\Gamma_{3PP} = \frac{2!3!\zeta(2)\zeta(3)(u+1)^2}{16\pi^2\rho_0 \aleph^5 c_0^6} (k_B T)^5 . \qquad (2.7)$$

As for the 4PP case, the direction of momenta of the colliding particles is not changed in the 5PP case. The calculation of the transition matrix elements by second-order perturbation theory is very complicated, and some

terms contain vanishing denominators under the condition  $\gamma$  = 0 in Eq. (1.1) and collinear scattering. The 5PP has maximum probablity in small-angle scattering and leads to equilibrium for the phonons in the direction of motion. Instead of these tedious calculations using second-order perturbation theory, we make use of the kinetic coefficient given by Landau and Khalatnikov<sup>15</sup> to obtain the  $\Gamma_{5PP}$  as 11

$$\Gamma_{\rm SPP} = aT^7 \quad . \tag{2.8}$$

Here, a is constant which can be determined experimentally by the attenuation coefficient of ultrasonic waves.

Now we consider the transformations of phonons into rotons and vice versa. When energetic phonons with energy on the order of  $\Delta$  collide with rotons  $(\vec{p}_1 + \vec{P}_2 \stackrel{\leftarrow}{\vdash} \vec{P}_3 + \vec{P}_4)$ , where  $\vec{p}$  and  $\vec{P}$  represent the phonon and roton momenta, respectively, the transformation can occur. Therefore, a phonon has at least a very large energy  $\Delta$ , and the interactions between phonons and rotons are very similar to that of the scattering between rotons. Adopting Landau and Khalatnikov's assumption, 15 we may take the interactions between a roton and phonon to be a  $\delta$ -function potential,

$$V = V_0 \delta(\vec{r}_1 - \vec{r}_2)$$
 , (2.9)

where  $v_0$  is an interaction constant, and  $\vec{r}_1$  and  $\vec{r}_2$  are the position vectors of the phonons and rotons, respectively.

The rate of change per unit time in the roton number due to the transformation of rotons into phonons is given by

$$\dot{N}_{r} = - \iiint \{n_{1}N_{2}(N_{3}+1)(N_{4}+1) - (n_{1}+1)(N_{2}+1)N_{3}N_{4}\} d\omega \frac{d\vec{p}_{1}d\vec{P}_{2}}{(2\pi k)^{4}}, \qquad (2.10)$$

where  $n_i$  and  $N_i$  are the distribution functions with momenta  $\vec{p}_i$  and  $\vec{P}_i$ , respectively.  $d\omega$  is the differential decay rate for the transformation of phonons and rotons with momenta  $\vec{p}_1$  and  $\vec{P}_2$  into two rotons with momenta  $\vec{P}_3$  and  $\vec{P}_4$  and is given by

$$d\omega = \frac{2\pi}{\aleph} |V_{IF}|^2 \delta(E_1 + E_2 - E_3 - E_4) \frac{dP_3 dP_4}{(2\pi \aleph)^4} , \qquad (2.11)$$

where  $|V_{IF}|$  is the transition matrix element, and  $E_i$  and  $E_j$  are the phonon and roton energies, respectively. Combining Eq. (2.10) with (2.11), we get

$$\dot{N}_r - \iiint \{n_1 N_2 (N_3 + 1) (N_4 + 1) - (n_1 + 1) (N_2 + 1) N_3 N_4\}$$

$$\times \frac{2\pi}{\aleph} |V_{1F}|^2 \delta(E_1 + E_2 - E_3 - E_4) \frac{d\vec{p}_1 d\vec{P}_2 d\vec{P}_3 d\vec{P}_4}{(2\pi \aleph)^8} . \tag{2.12}$$

Now we construct the symmetrized pairwise plane wave over incoming and outgoing phonons and rotons as

$$\psi(\vec{p}_{1},\vec{P}_{2}) = \frac{1}{\sqrt{2}} \left[ e^{\frac{i}{M}(\vec{p}_{1} \cdot \vec{r}_{1} + \vec{P}_{2} \cdot \vec{r}_{2})} + e^{\frac{i}{M}(\vec{p}_{1} \cdot \vec{r}_{2} + \vec{P}_{2} \cdot \vec{r}_{1})} \right] ,$$

$$\psi(\vec{P}_{3},\vec{P}_{4}) = \frac{1}{\sqrt{2}} \left[ e^{\frac{i}{M}(\vec{P}_{3} \cdot \vec{r}_{1} + \vec{P}_{4} \cdot \vec{r}_{2})} + e^{\frac{i}{M}(\vec{P}_{3} \cdot \vec{r}_{2} + \vec{P}_{4} \cdot \vec{r}_{1})} \right] ,$$

$$(2.13)$$

and combining Eq. (2.13) with (2.11), we can obtain the transition matrix elements as

$$v_{IF} = 2v_0 \Omega^{-\frac{1}{2}} \int e^{\frac{i}{p}(\vec{P}_3 + \vec{P}_4 - \vec{p}_1 - \vec{P}_2) \cdot \vec{r}_1} d\Omega_1 \qquad (2.14)$$

Here,  $d\Omega$  represents the area element. Making use of the  $\delta$ -function identity together with Eq. (2.14), we obtain

$$\delta(\vec{P}_3 + \vec{P}_4 - \vec{P}_1 - \vec{P}_2) = \frac{1}{(2\pi \aleph)^2} \int e^{\frac{i}{\aleph}(\vec{P}_3 + \vec{P}_4 - \vec{P}_1 - \vec{P}_2) \cdot \vec{r}_1} d\Omega_1 , \qquad (2.15)$$

and performing the integration over momentum space, we get

$$\int |v_{IF}|^2 \frac{d\vec{P}_2}{(2\pi\hbar)^2} - 4|v_0|^2 . \qquad (2.16)$$

As mentioned earlier, from the fact that the roton momentum  $\vec{P}$  is almost the same as that of the roton parameter  $\vec{P}_0$ , the expansion of the roton distribution function N as a function of chemical potential yields Eq. (2.12) as

$$\dot{N}_{r} = \frac{\mu_{r} - \mu_{ph}}{k_{B}T} \frac{16\pi^{2}}{\cancel{N}} \frac{\Delta |V_{0}|^{2}}{c_{0}^{2}} \frac{1}{(2\pi\cancel{N})^{6}} \iint N_{30}N_{40} d\vec{P}_{3} d\vec{P}_{4} . \qquad (2.17)$$

where  $\mu_r$  and  $\mu_{ph}$  are the roton and phonon chemical potentials, respectively, and performing the integration over momentum space we arrive at

$$\dot{N}_{r} = \frac{\mu_{r} - \mu_{ph}}{k_{B}T} \frac{4\Delta |V_{0}|^{2} N_{r}^{2}}{k_{O}^{3} c_{O}^{2}} , \qquad (2.18)$$

where  $N_{r}$  is the number density of roton given by

$$N_{r} = \left(\frac{\mu k_{B}T}{2\pi}\right)^{\frac{1}{2}} \frac{P_{0}}{\mu^{2}} e^{-\Delta/k_{B}T} \qquad (2.19)$$

The rate of change per unit time in the roton number can be expressed in terms of the kinetic coefficient  $\Gamma_{\rm ph-r}$  as

$$\dot{N} = \Gamma_{\text{ph-r}} (\mu_{\text{r}} - \mu_{\text{ph}}) \quad . \tag{2.20}$$

Comparing Eqs. (2.20) and (2.18), we can deduce the kinetic coefficient  $\Gamma_{\rm ph}$ -r for the transformation of rotons into phonons and vice versa as

$$\Gamma_{\text{ph-r}} = \frac{4\Delta |v_0|^2 N_r^2}{\mu^3 c_0^2 k_B^T} \quad . \tag{2.21}$$

Since  $V_0$  is not known experimentally, we can simply rewrite Eq. (2.21) as

$$\Gamma_{\text{ph-r}} = b e^{-2\Delta/k_B T} \qquad (2.22)$$

where b is a constant which contains several parameters of the elementary excitation and can be determined experimentally by the ultrasonic attenuation.

## 3. Coefficients of two-dimensional second viscosity

When liquid  $^4\mathrm{He}$  is in its equilibrium state, the distribution functions for phonons and rotons are expressed by

$$n_{\text{Oph}} = \left[\exp\left(\frac{E_{p} - \vec{p} \cdot (\vec{v}_{s} - \vec{v}_{n})}{k_{p}T}\right) - 1\right]^{-1}$$
, (3.1)

$$n_{0r} = \left[ \exp\left(\frac{E_r - \vec{p} \cdot (\vec{v}_n - \vec{v}_s)}{k_B T}\right) \right]^{-1}$$
 (3.2)

However, due to the presence of the energy dissipation, nonequilibrium effects appear in energy and momentum conservations, superfluid flow, energy flux, and especially entropy which is not conserved but increases (entropy increases in order to determine the unknown dissipation coefficients). Taking into account all of these conditions, we can accumulate all these effects within the following two hydrodynamic equations which concern

$$\frac{\partial \vec{j}}{\partial t} + \vec{\nabla}P + \vec{v}_{S}\vec{\nabla}\vec{j} + (\vec{j} - \rho\vec{v}_{S})\vec{\nabla}\vec{v}_{n} + (\vec{j} \cdot \vec{\nabla})\vec{v}_{S} + (\vec{v}_{n} \cdot \vec{\nabla})(\vec{j} - \rho\vec{v}_{S})$$

$$= \vec{\nabla}\{\zeta_{1}\vec{\nabla}(\vec{j} - \rho\vec{v}_{m}) + \zeta_{2}\vec{\nabla}\vec{v}_{n}\} , \qquad (3.3)$$

$$\frac{\partial \vec{\mathbf{v}}}{\partial t} + \vec{\nabla}(\mu + \frac{\vec{\mathbf{v}}^2}{2}) = \vec{\nabla}(\zeta_3 \vec{\nabla}(\vec{\mathbf{j}} - \rho \vec{\mathbf{v}}_n) + \zeta_4 \vec{\nabla} \vec{\mathbf{v}}_n) . \tag{3.4}$$

Here,  $\vec{j}$  and  $\rho$  are the momentum and mass density, P is the pressure,  $\mu$  is the chemical potential of the liquid helium, and  $\varsigma_1$ ,  $\varsigma_2$ ,  $\varsigma_3$  and  $\varsigma_4$  are the

coefficients of second viscosity. The coefficients  $\varsigma_1$  and  $\varsigma_4$  are equal according to the Onsager's reciprocity principle.

It is obvious that the coefficients of the second viscosity depend on the variations of the temperature and thermodynamic functions, and thus the number of phonons and rotons are changed by the various processes that we have mentioned earlier. Let  $N_{ph}$  and  $N_{r}$  be the number of phonons and rotons per unit area, respectively, and  $\mu_{ph}$  and  $\mu_{r}$  be their chemical potentials. When the system deviates slightly from its equilibrium state,  $N_{r}$  and  $N_{ph}$  are changed in time and try to return to their values in the equilibrium state. We assume that the nonequilibrium distribution function n deviates very slightly from equilibrium, i.e., small deviations of the density and entropy can be determined by the time derivatives of the phonon and roton numbers,  $\mathring{N}_{ph}$  and  $\mathring{N}_{r}$ , in the expansion of the chemical potential. Since the phonons and rotons take part in normal motion with velocity  $\overset{\rightarrow}{v}_{n}$ , neglecting the quadratic effects and taking only linear terms in  $\mu_{ph}$  and  $\mu_{r}$ , we can obtain the following equations:

$$\dot{N}_{r} + N_{r} \overrightarrow{\nabla v}_{n} = -\gamma_{rr} \mu_{r} + \gamma_{ph-r} \mu_{ph} , \qquad (3.5)$$

$$\mathring{N}_{ph} + \mathring{N}_{ph} \overrightarrow{\nabla v}_{n} = \gamma_{ph-r} - \gamma_{pp} \mu_{ph} . \qquad (3.6)$$

The kinetic coefficients on the right-hand sides of Eq. (3.5)-(3.6) are symmetric in the indices r and ph.

Since the rate of change per unit time in the density ho and entropy S can be expressed by the continuity equations

$$\dot{S} + \vec{\nabla} S \vec{v}_{m} = 0 \quad , \tag{3.8}$$

substitution of Eqs. (3.7)-(3.8) into (3.5)-(3.6) vields

$$\dot{N}_{r} = -\frac{\partial N_{r}}{\partial \rho} \overrightarrow{\nabla j} - \frac{\partial N_{r}}{\partial S} \overrightarrow{\nabla v}_{n} + \frac{\partial N_{r}}{\partial \rho} S \overrightarrow{\nabla} \rho \overrightarrow{v}_{n} - \frac{\partial N_{r}}{\partial \rho} \rho \overrightarrow{\nabla v}_{n} , \qquad (3.9)$$

$$\dot{N}_{ph} = \frac{\partial N_{ph}}{\partial \rho} \overrightarrow{\nabla j} - \frac{\partial N_{ph}}{\partial S} S \overrightarrow{\nabla v}_{n} + \frac{\partial N_{ph}}{\partial \rho} \overrightarrow{\nabla} \rho \overrightarrow{v}_{n} - \frac{\partial N_{ph}}{\partial \rho} \rho \overrightarrow{\nabla v}_{n}$$
(3.10)

Combining Eqs. (3.9)-(3.10) with Eqs. (3.5)-(3.6), we can express  $\mathring{N}_r$  and  $\mathring{N}_{ph}$  in terms of  $\rho$  and S by

$$-\frac{\partial N_{r}}{\partial \rho} \vec{\nabla} (\vec{j} - \rho \vec{v}_{n}) + (N_{r} - \frac{\partial N_{r}}{\partial S} S - \frac{\partial N_{r}}{\partial \rho} \rho) \vec{\nabla} \vec{v}_{n} = -\gamma_{rr} \mu_{r} + \gamma_{ph-r} \mu_{ph} , \quad (3.11)$$

$$-\frac{\partial N_{ph}}{\partial \rho} \vec{\nabla} (\vec{j} - \rho \vec{v}_n) + (N_{ph} - \frac{\partial N_{ph}}{\partial S} S - \frac{\partial N_{ph}}{\partial \rho} \rho) \vec{\nabla} \vec{v}_n - \gamma_{ph-r} \mu_r - \gamma_{pp} \mu_{ph} . \quad (3.12)$$

The kinetic coefficients  $\gamma_{\rm rr}$ ,  $\gamma_{\rm ph-r}$  and  $\gamma_{\rm pp}^{15,17}$  may be replaced as  $\Gamma_{\rm 3rp}$ ,  $\Gamma_{\rm 5pp}$ ,  $\Gamma_{\rm ph-r}$  and  $\Gamma_{\rm 3pp}$ , which are the kinetic coefficients of the three-roton process (3RP), 5PP, phonon-roton interaction and 3PP, respectively, as follows

$$\gamma_{rr} = \Gamma_{3rp} + \Gamma_{ph-r}$$
,

$$\gamma_{r-ph} = \gamma_{ph-r} = \Gamma_{ph-r} , \qquad (3.13)$$

$$\gamma_{pp} = \Gamma_{3pp} + \Gamma_{5pp} + \Gamma_{ph-r}$$
.

Since  $\Gamma_{\rm 3rp}$  is very small compared to the other kinetic coefficients, <sup>17</sup> we may neglect it. Substituting Eq. (3.13) into Eqs. (3.11)-(3.12) and solving for  $\mu_{\rm ph}$  and  $\mu_{\rm r}$ , we obtain

$$\mu_{\text{ph}} = \frac{1}{\Gamma_{3\text{PP}} + \Gamma_{5\text{PP}}} \left( \frac{\partial N}{\partial \rho} \vec{\nabla} (\vec{j} - \rho \vec{v}_{n}) - (N - \frac{\partial N}{\partial S} S - \frac{\partial N}{\partial \rho} \rho) \vec{\nabla} \vec{v}_{n} \right) , \qquad (3.14)$$

$$\mu_{\rm r} = \frac{1}{\Gamma_{\rm 3PP} + \Gamma_{\rm 5PP}} \left\{ \frac{\partial N}{\partial \rho} \; \vec{\nabla} (\vec{\rm j} - \rho \vec{\rm v}_{\rm n}) - (N - \frac{\partial N}{\partial S} S - \frac{\partial N}{\partial \rho} \rho) \; \vec{\nabla} \vec{\rm v}_{\rm n} \right\}$$

$$+\frac{1}{\Gamma_{\text{ph-r}}} \left\{ \frac{\partial N_{\text{r}}}{\partial \rho} \vec{\nabla} (\vec{j} - \rho \vec{v}_{\text{n}}) - (N_{\text{r}} - \frac{\partial N_{\text{r}}}{\partial S} S - \frac{\partial N_{\text{r}}}{\partial \rho} \rho) \vec{\nabla} \vec{v}_{\text{n}} \right\} , \qquad (3.15)$$

where  $N = N_r + N_{ph}$ .

In Eqs. (3.3)-(3.4) the pressure P and chemical potential  $\mu$  can be expressed in terms of the chemical potentials  $\mu_{\rm ph}$  and  $\mu_{\rm r}$  as

$$\vec{\nabla}P = - \vec{\nabla}(\frac{\partial P}{\partial \mu_{r}}\mu_{r} + \frac{\partial P}{\partial \mu_{ph}}\mu_{ph}) ,$$

$$\vec{\nabla}\mu = - \vec{\nabla}(\frac{\partial \mu}{\partial \mu_{r}}\mu_{r} + \frac{\partial \mu}{\partial \mu_{ph}}\mu_{ph}) .$$
(3.16)

Solving Eqs. (3.14)-(3.16) together with Eqs. (3.3)-(3.4) for the coefficients  $\zeta_1$ ,  $\zeta_2$ ,  $\zeta_3$  and  $\zeta_4$  of the second viscosity, we obtain

$$\varsigma_{1}(T) = -\frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \left( \frac{\partial P}{\partial \mu_{r}} + \frac{\partial P}{\partial \mu_{ph}} \right) \frac{\partial N}{\partial \rho} - \frac{1}{\Gamma_{ph-r}} \frac{\partial P}{\partial \mu_{r}} \frac{\partial N_{r}}{\partial \rho} ,$$
(3.17)

$$\varsigma_{2}(T) = \frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \left( \frac{\partial P}{\partial \mu_{r}} + \frac{\partial P}{\partial \mu_{ph}} \right) \left( N - \frac{\partial N}{\partial S} S - \frac{\partial N}{\partial \rho} \rho \right) + \frac{1}{\Gamma_{ph-r}} \frac{\partial P}{\partial \mu_{r}}$$

$$\times \left(N_{r} + \frac{\partial N_{r}}{\partial S}S - \frac{\partial N_{r}}{\partial \rho}\rho\right) , \qquad (3.18)$$

$$\zeta_{3}(T) = -\frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \left( \frac{\partial \mu}{\partial \mu_{r}} + \frac{\partial \mu}{\partial \mu_{ph}} \right) \frac{\partial N}{\partial \rho} - \frac{1}{\Gamma_{ph-r}} \frac{\partial \mu}{\partial \mu_{r}} \frac{\partial N_{r}}{\partial \rho} . \tag{3.19}$$

$$\zeta_{4}(T) = \frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \left( \frac{\partial \mu}{\partial \mu_{r}} + \frac{\partial \mu}{\partial \mu_{ph}} \right) \left( N - \frac{\partial N}{\partial S} S - \frac{\partial N}{\partial \rho} \rho \right) + \frac{1}{\Gamma_{ph-r}} \frac{\partial \mu}{\partial \mu_{r}}$$

$$\times \left(N_{r} - \frac{\partial N_{r}}{\partial S}S - \frac{\partial N_{r}}{\partial \rho}\rho\right) . \tag{3.20}$$

Making use of the thermodynamic identities

$$dE_0 = TdS + \mu d\rho - N_r d\mu_r - N_{ph} d\mu_{ph} , \qquad (3.21)$$

$$P = -E_0 + ST + \mu \rho$$
 , (3.22)

we can transform Eqs. (3.17)-(3.20) into the following forms:

$$\varsigma_{1}(T) = \frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \frac{\partial N}{\partial \rho} (N - \frac{\partial N}{\partial S}S - \frac{\partial N}{\partial \rho}\rho) - \frac{1}{\Gamma_{ph-r}} \frac{\partial N_{r}}{\partial \rho} (N_{r} - \frac{\partial N_{r}}{\partial S}S - \frac{\partial N_{r}}{\partial \rho}\rho) ,$$
(3.23)

$$\varsigma_{2}(T) = \frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \left(N - \frac{\partial N}{\partial S}S - \frac{\partial N}{\partial \rho}\rho\right)^{2} + \frac{1}{\Gamma_{ph-r}} \left(N_{r} - \frac{\partial N_{r}}{\partial S}S - \frac{\partial N_{r}}{\partial \rho}\rho\right) , \quad (3.24)$$

$$\varsigma_3(T) = \frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \left( \frac{\partial N}{\partial \rho} \right)^2 + \frac{1}{\Gamma_{ph-r}} \left( \frac{\partial N_r}{\partial \rho} \right)^2$$
 (3.25)

$$\zeta_4(T) = -\frac{1}{\Gamma_{3PP} + \Gamma_{5PP}} \frac{\partial N}{\partial \rho} (N - \frac{\partial N}{\partial S} S - \frac{\partial N}{\partial \rho} \rho) - \frac{1}{\Gamma_{ph-r}} \frac{\partial N_r}{\partial \rho}$$

$$\times (N_{r} - \frac{\partial N_{r}}{\partial S}S - \frac{\partial N_{r}}{\partial \rho}\rho) = \zeta_{1}(T) . \qquad (3.26)$$

Here,  $\zeta_1$  and  $\zeta_4$  are shown to be equal as expected from Onsager's reciprocity symmetry relation for the kinetic coefficients.

As we have recently proven, near T=0 K the 3PP are the main influence on the first viscosity coefficient in  $2D^{11}$  and  $3D^{12}$  liquid helium, which was not shown by Landau and Khalatnikov, and in ultrasonic sound attenuation the 3PP also influence the coefficients of second viscosity near T=0 K. Therefore, to investigate the contribution of the 3PP to the coefficients of the second viscosity, we should consider the 3PP effects separately from other processes. To do this, we only keep the terms for the 3PP in Eqs. (3.3)-(3.12) and then obtain

$$\mu_{\rm ph} = \frac{1}{\Gamma_{\rm 3PP}} \left\{ \frac{\partial N_{\rm ph}}{\partial \rho} \vec{\nabla} (\vec{j} - \rho \vec{v}_{\rm n}) - (N_{\rm ph} - \frac{\partial N_{\rm ph}}{\partial S} S - \frac{\partial N_{\rm ph}}{\partial \rho} \rho) \vec{\nabla} \vec{v}_{\rm n} \right\} . \tag{3.27}$$

The pressure P and chemical potential  $\mu$  in Eqs. (3.3)-(3.4) and Eq. (3.6), which depend only on the phonon chemical potential, can be written as

$$\vec{\nabla}P = -\vec{\nabla}\left[\frac{\partial P}{\partial \mu_{ph}} \mu_{ph}\right] , \qquad \vec{\nabla}\mu = -\vec{\nabla}\left[\frac{\partial \mu}{\partial \mu_{ph}} \mu_{ph}\right] . \qquad (3.28)$$

Therefore, with the use of the thermodynamic identities Eqs. (3.21)-(3.22), we can obtain the contributions from the 3PP to the coefficients of the second

viscosity in Eq. (3.23)-(3.26), together with Eqs. (3.27)-(3.28), near T = 0 K as

$$\varsigma_1(T) = -\frac{1}{\Gamma_{3PP}} \frac{\partial N_{ph}}{\partial \rho} \left[ N_{ph} - \frac{\partial N_{ph}}{\partial S} S - \frac{\partial N_{ph}}{\partial \rho} \rho \right] ,$$
 (3.29)

$$\varsigma_2(T) = \frac{1}{\Gamma_{3PP}} \left[ N_{ph} - \frac{\partial N_{ph}}{\partial S} S - \frac{\partial N_{ph}}{\partial \rho} \rho \right]^2$$
, (3.30)

$$\varsigma_3(T) = \frac{1}{\Gamma_{3pp}} \left( \frac{\partial N_{ph}}{\partial \rho} \right)^2 ,$$
(3.31)

$$\varsigma_4(T) = -\frac{1}{\Gamma_{3PP}} \frac{\partial N_{ph}}{\partial \rho} \left( N_{ph} - \frac{\partial N_{ph}}{\partial S} S - \frac{\partial N_{ph}}{\partial \rho} \rho \right) .$$
 (3.32)

Here,  $\varsigma_1$  and  $\varsigma_4$  are equal and thus satisfy Onsager's reciprocity symmetric principle.

#### 4. Results and discussion

In the previous sections we have derived the kinetic coefficients for two temperature regions, and making use of these coefficients we have obtained the four coefficients of the second viscosity. For  $T \leq 0.3$  K the contributions of the 3PP to the coefficients are most important, and for  $T \geq 0.3$  K we may neglect the contribution from the 3PP. Therefore, dropping the  $\Gamma_{\rm 3PP}$  terms in Eqs. (3.23)-(3.26), these coefficients of the two-dimensional (2D) second viscosity are reduced to those of the bulk case. <sup>16</sup> except for differences between 2D and 3D thermodynamic dimensionality. Therefore, we will not write the coefficients here repeatedly for T > 0.3 K.

To analyze concretely the four coefficients of the 2D second viscosity for  $T \leq 0.3$  K, we use the following two-dimensional thermodynamic functions:  $^8$ 

$$s_{ph}(T) = \frac{k_B}{2\pi} \left( \frac{3\zeta(3)}{\mu^2 c_0^2} (k_B T)^2 + \frac{5!\zeta(5)B_1}{c_0^5} (k_B T)^4 + \dots \right) , \qquad (4.1)$$

$$s_r(T) = \left(\frac{\mu_0 k_B^T}{2 \pi \hbar^2}\right)^{k_2} \frac{k_B^P o}{\hbar} \left(\frac{3}{2} + \frac{\Delta}{k_B^T}\right) e^{-\Delta/k_B^T},$$
 (4.2)

$$N_{\text{ph}}(T) = \frac{\zeta(2)}{2\pi} \left(\frac{k_{\text{B}}T}{k_{\text{C}}}\right)^2 \qquad (4.3)$$

Here,  $B_1 = \{1 + \frac{3\pi na^2}{16} (\frac{3}{2}a - v_0)\}/2c_0$ , n is the number density of 2D liquid helium, and the other parameters are given in Eq. (1.3). To see the temperature variation of the coefficients numerically, we adopt the parameters which are deduced from the specific data of Bretz et al. Because To explain the various experimental data,  $^{9,10}$  we have made use of these parameters successfully, which are listed in Table I, where  $q_0 = P_0/M$ . With this choice, we have obtained the sound velocity  $c_0 = 164.4$  m/s at absolute zero temperature. However, from the analysis of the sound attenuation, we obtained  $c_0 = 84.06$  m/s, which is very closa to the experimental value of  $76 \pm 2$  m/s given by Wushburn et al. And the parameters u, a and b in Eqs. (2.7), (2.8) and (2.22) are assumed to be 1.8, (20.8) and (2.22) are assumed to be (20.8)0. With the use of the above parameters and those in Table I, the numerical expressions for the kinetic coefficients are given by

$$\Gamma_{3PP} = 1.24 \times 10^{39} \text{ T}^5$$
 , (4.4)

$$\Gamma_{5pp} = 1.0 \times 10^{43} \text{ T}^7$$
 , (4.5)

$$\Gamma_{\text{ph-r}} = 4 \times 10^{49} \, e^{-8.24/T}$$
 (4.6)

Through Eqs. (4.4)-(4.6) we see that the powers of the temperatures in the kinetic coefficients vary according to the interactions between elementary excitations.

Substituting Eqs. (4.1)-(4.4) into Eqs. (3.29)-(3.32), we obtain the temperature variation of the four coefficients of the 2D second viscosity as follows:

$$\varsigma_{1}(T) = 4.32 \times 10^{-4} T^{-1} \left[1 - \frac{2\left(\left(\frac{3}{2} + \frac{4.12}{T}\right) + 3.59 \times 10^{-2} T^{3/2} e^{4.12/T}\right)}{\left(\frac{3}{4} + \frac{4.12}{T} + \frac{16.97}{T^{2}}\right) + 7.19 \times 10^{-2} T^{3/2} e^{4.12/T}} + 3.6\right],$$
(4.7)

$$5_{2}(T) = 2.23 \times 10^{-13} T^{-1} [1 - \frac{2((\frac{3}{2} + \frac{4.12}{T}) + 3.59 \times 10^{-2} T^{-3/2} e^{4.12/T})}{(\frac{3}{4} + \frac{4.12}{T} + \frac{16.97}{T^{2}}) + 7.19 \times 10^{-2} T^{3/2} e^{4.12/T}} + 3.6]^{2},$$
(4.8)

$$\varsigma_3(T) = 8.39 \times 10^5 T^{-1}$$
 (4.9)

$$\zeta_{4}(T) = \zeta_{1}(T) \qquad (4.10)$$

When T drops to less than 0.3 K, the second (fractional) term inside the square brackets in Eqs. (4.7) and (4.8) approaches unity, and these equations are reduced to the following two equations, respectively:

$$\varsigma_1(T) = 1.555 \times 10^{-3} T^{-1}$$
, (4.11)

$$\varsigma_2(T) = 2.890 \times 10^{-12} T^{-1}$$
 (4.12)

All four coefficients have a T<sup>-1</sup>-dependence, which is the same as that of the coefficient of the first viscosity. Here, we note that there are generally thirteen independent dissipative coefficients, <sup>22,23</sup> but we only consider the above five coefficients. Moreover, we will not discuss the pressure and frequency variations of the coefficients of the second viscosity.

We have already shown that the 2D thermal conductivity and first viscosity of thin liquid helium films  $^{11}$  are all positive. We can easily confirm that the four coefficients of second viscosity are all positive. For T = 0.05 K we find that  $\zeta_1^2 < \zeta_2 \zeta_3$ , where  $\zeta_1^2 - 10^{-3}$  and  $\zeta_2 \zeta_3 \sim 10^{-2.7}$ . However, as the temperature increases to about 0.1 K, the equality then holds, i.e.,  $\zeta_1^2 = \zeta_2 \zeta_3$ . Comparing the order of magnitude for the four coefficients, we can write  $\zeta_3 > \zeta_1 > \zeta_2$ , and all four coefficients have larger values than that of the coefficient of the first viscosity.

In conclusion, the coefficients of the second viscosity in two-dimensional thin helium films behave like those of bulk liquid helium for  $T \geq 0.3$  K, while for  $T \leq 0.3$  K, the four coefficients of the second viscosity exhibit a  $T^{-1}$ -dependence like that of the first viscosity, which is due to the 3PP originating from the anomalous excitation spectrum of two-dimensional liquid helium at low momenta and low temperatures.

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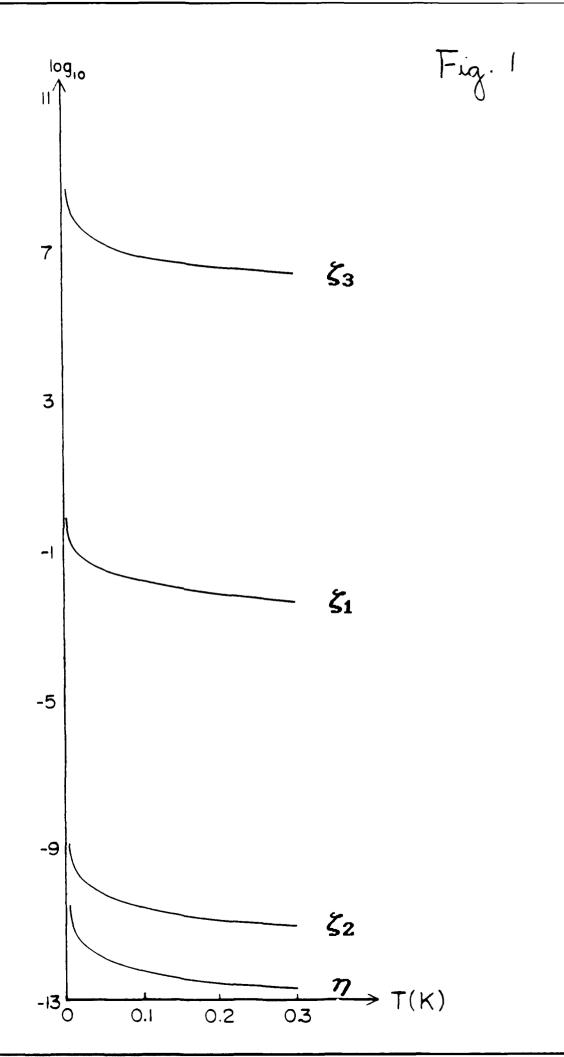
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Table I. Theoretical parameters.

n (Å <sup>-2</sup> )	$\Delta/\mathbf{k}_{B}$ (K)	q <sub>0</sub> (Å <sup>-1</sup> )	<sup>μ</sup> 0	c <sub>0</sub> (m/s)
$2.79 \times 10^{-2}$	4.12	1.02	0.75 m <sub>.</sub>	164.4

# Figure Caption

Figure 1. Temperature variation of the coefficients  $\zeta_1$ ,  $\zeta_2$ ,  $\zeta_3$  and  $\zeta_4$  of the second viscosity and the coefficient  $\eta$  of the first viscosity in thin helium film.



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